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Study on the Eulerian dispersed phase equations in non-uniform turbulent two-phase flows: discussion and comparison with experiments

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Abstract

The work presented here analyses the underlying mechanisms in the continuum equations for non-colliding particles in dilute non-uniform two-phase flows. The study is carried out for axisymmetric particle-laden gas jets, which constitute a good example of non-uniform flow. Equations of radial and axial momentum and turbulent kinetic energy for the particulate phase are divided into their basic terms and analysed separately. As a result, for high inertia particles, the modelling of the 'interaction terms' (terms due to gas–particle interaction) reveals itself as the crucial point, as long as they drive the existing equilibrium in the Eulerian particle equations. The hypotheses used to model the dispersed phase Reynolds stresses are in accordance with the previous theoretical work of Reeks (Reeks, M.W., 1993. On the constitutive relations for dispersed particles in non-uniform flows I: Dispersion in a simple shear flow. Phys. Fluids A 5, 750–761): the normal stresses in the streamwise direction are enhanced over the corresponding ones in the other directions and the former prevail over the shear stresses in the limit of great inertia particles, too. This particle Reynolds stress modelling is found to be directly related with the correct prediction of the dispersion of the corresponding volume fraction profile, α^d, in the considered experiments. Moreover, the information obtained from the balance of present contributions in the radial momentum equation confirms a previously used closure of radial relative velocity as proportional to the gradient of particle void fraction (Ishii, M., 1975. Thermo-fluid dynamic theory of two-phase flow, Eyrolles; Lee, S.L., Wiesler, M.A., 1986. Theory on transverse migration of particles in a turbulent two-phase suspension flow due to turbulent diffusion. Int. J. Multiphase Flow 12, 99–111; Simonin, O., 1990. Eulerian formulation for particle dispersion in turbulent two-phase flows. In: Proceedings of the 5th Workshop on Two-Phase Flow Predictions, Erlangen, Germany). This effect is the ultimate responsible for the spreading of α^d along the jet. Comparisons of numerical calculations with the experiments of Mostafa et al. (Mostafa, A.A., Mongia, H.C., McDonell, V.G., Samuelsen, G.S., 1989. Evolution of particle laden jet flows: A theoretical and experimental study. AIAA J. 27, 167–183) are provided showing reasonable agreement in all available variables, included the particulate normal Reynolds stresses.

Keywords: Non-uniform turbulent two-phase flow; Particle Reynolds stresses; Anisotropy

Notation

C_D	inverse particle relaxation time
C	constant of the model
d_p	particle diameter
D	nozzle diameter
$\boldsymbol{g}$	acceleration of gravity
I^x	interaction term
k	kinetic turbulent energy
P	average pressure
r	radial coordinate
Re_p	particle Reynolds number
$\boldsymbol{u}$	instantaneous gas velocity
$\boldsymbol{V}$	average particle velocity
$\boldsymbol{W}$	average relative velocity
x	axial coordinate
X	axial distance from the nozzle
Greeks	
α	void fraction
δ	Dirac's delta
ϵ	dissipation rate
μ	viscosity
ρ	density
σ	Schmidt number
τ	time scale

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υ	instantaneous particle velocity
ϕ	generic variable
φ	Azimuthal coordinate
χ	mass loading ratio
$\mathscr{P}$	production of k
Subscripts	
L	Lagrangian scale
∞	long term behaviour
Superscripts	
$'$	fluctuating component
d	dispersed phase
m	average mixture property

1. Introduction

Nowadays, two approaches are mainly used to describe the dispersed phase in a two-phase flow (solid, droplet or bubble suspensions). In the so-called *Lagrangian* method the discrete elements are tracked through a random fluid field by solving their equations of motion. In the second methodology, both phases are handled as two interpenetrating continuums and are governed by a set of differential equations representing conservation laws; this approach is named as *Eulerian*. In this last context, for establishing the dispersed elements equations, two possibilities come out. First, the second phase is considered as a fluid for all effects. This corresponds to the well-known two-fluid model. Second, the non-continuous phase is thought of as a cloud of material elements, whose behaviour is driven by a probability density function (PDF), depending on each element variables, that responds to a kinetic transport equation similar to the Maxwell–Boltzmann one. The continuum equations for the second phase are obtained as the statistical moments of such PDF-evolution equation.

In spite of the lack of a complete agreement about the final form of the equations and the constitutive relations used for the dispersed phase, the Eulerian strategies continue to be attractive from an engineering point of view because of their simplicity and computational efficiency.

However, the traditional closures, even giving approximated values for the mean fields, fail in the predictions of particle turbulent quantities specially in non-uniform flows. To overcome this fact considerable effort has been devoted during the last years to develop turbulence closures at the level of second moments of the particulate phase (Reeks, 1993; Hyland et al., 1998; Février and Simonin, 1998; Zaichik, 1997), but those are still in the research stage. Following this investigation direction, the relevant underlying mechanisms in the discrete element continuum equations in a dilute non-uniform gas–solids jet flow is identified as one objective pursued in this paper. To achieve this end, the momentum and fluctuating energy equations are divided into their basic terms and analysed separately. We will show the importance of modelling the normal stresses and it is found that they are directly implied in the spreading of the particles along the jet. A collateral result is the confirmation of a closure for the radial relative drift velocity introduced previously by some authors (Ishii, 1975; Wiesler, 1986; Simonin, 1990).

The employed modelling of particulate Reynolds stresses is consistent with previous experimental (Mostafa et al., 1989; Hishida and Maeda, 1997) and theoretical (Reeks, 1993; Zaichik, 1997) works, which establish in the case of great inertia dispersed elements, that the normal stresses in the streamwise direction dominate over the stresses in the other directions and over the shear stresses, too. In addition, a Boussinesq closure for these last stresses can be adopted in the former limit by means of using the concept of fluctuating diffusivity coefficient that is formulated here as proportional to the particle mean square velocity in the required direction and its response time.

The predictions are compared with the measurements of Mostafa et al. (1989) providing a reasonable agreement in all available mean quantities. The good behaviour of the dispersed phase void fraction and fluctuating kinetic energy is remarkable. These quantities are directly related to the evolution of the normal Reynolds stresses, which can be highly anisotropic. For this reason, the predicted particle normal stresses are shown versus the previous experiments and also versus those of Hishida and Maeda (1997). These comparisons manifest the crucial role played by the particulate Reynolds stresses in order to reach accurate predictions in non-uniform two-phase flows.

Section 2 present the basic averaged equations, for the continuous as well as for the dispersed phase. Section 3 deals with the modelling of the turbulence of the dispersed phase and the related closures are introduced. In Section 4, the dispersed phase equations are split in their constitutive terms and are analysed separately and the emerging results discussed. The comparison with the experimental measurements is found in Section 5, and Section 6 addresses the conclusions and future work.

2. Governing equations

Using the Dispersed Elements PDF – Indicator Function ensemble conditioned average (Aliod and Dopazo, 1990; Prosperetti and Zhang, 1994), the following Eulerian model equations for both phases, gas and solids, in the context of isothermal dilute flows, are considered (the superscript d refers to the dispersed phase):

Gas phase. Mass conservation equation

$$[\rho\alpha]_{,t} + [\rho\alpha U_i]_{,i} = 0. \tag{1}$$

Momentum conservation equation

$$\begin{aligned}[\rho\alpha U_j]_{,t} + [\rho\alpha U_i U_j]_{,i} = &-\left[\left[P + \frac{2}{3}\alpha(\rho k + \mu_T U_{k,k})\right]\delta_{ij}\right]_{,i} \\ &+ [\alpha(\mu + \mu_T)[U_{i,j} + U_{j,i}]]_{,i} \\ &- I_j^D + f_j^V \end{aligned} \tag{2}$$

Fluctuating kinetic energy equation k

$$[\rho\alpha k]_{,t} + [\rho\alpha k U_i]_{,i} = \left[\alpha\left(\mu + \frac{\mu_T}{\sigma_k}\right)k_{,i}\right]_{,i} + \alpha[\mathscr{P} - \rho\epsilon] + I^{WD} - I^W. \tag{3}$$

Dissipation rate of turbulent kinetic energy equation ϵ:

$$\begin{aligned}[\rho\alpha\epsilon]_{,t} + [\rho\alpha\epsilon U_j]_{,j} = &\left[\alpha\left(\mu + \frac{\mu_T}{\sigma_\epsilon}\right)\epsilon_{,j}\right]_{,j} \\ &+ \alpha\frac{\epsilon}{k}[C_{\epsilon_1}\mathscr{P} - C_{\epsilon_2}\rho\epsilon] + I^\epsilon. \end{aligned} \tag{4}$$

Dispersed phase. Mass conservation equation

$$[\rho^d\alpha^d]_{,t} + [\rho^d\alpha^d V_i]_{,i} = 0. \tag{5}$$

Momentum conservation equation

$$[\rho^d\alpha^d V_j]_{,t} + [\rho^d\alpha^d V_i V_j]_{,i} = \left[-\alpha^d\rho^d\overline{\upsilon_j'\upsilon_i'}^{\,d}\right]_{,i} + I_j^D + f_j^{dV}. \tag{6}$$

Fluctuating kinetic energy equation $k^d = \overline{k'^d}^{\,d} = \frac{1}{2}\overline{\upsilon_i'\upsilon_i'}^{\,d}$

$$[\rho^d\alpha^d k^d]_{,t} + [\rho^d\alpha^d V_j k^d]_{,j} = \left[-\alpha^d\rho^d\overline{k'^d\upsilon_j'}^{\,d}\right]_{,j} + \alpha^d\mathscr{P}^d + I^W. \tag{7}$$

The conditioned ensemble averaged main variables are the velocities of continuous and dispersed phases, $\boldsymbol{U}$ and $\boldsymbol{V}$, the respective volume fractions, α and α^{d}, and the pressure, P. The respective velocity fluctuations are defined with respect to $\boldsymbol{U}$, $\boldsymbol{V}$ and are denoted as $\boldsymbol{u}'$, $\boldsymbol{v}'$. The densities of fluid phase, ρ, and solids, ρ^{d}, are supposed to be constant and μ denotes the fluid viscosity.

In the momentum Eqs. (2) and (6), I_j^D is the interaction term due to the aerodynamic drag, which is modelled as proportional to the ensemble averaged relative velocity

$$I_j^D = C_D\alpha^{\mathrm{d}}\rho^{\mathrm{d}}\left(U_j - V_j\right). \tag{8}$$

Spherical particles with constant diameter, d_{p}, are considered. Therefore, a standard drag law is used with $C_D = 18\ \mu f(Re_{\mathrm{p}})/(\rho^{\mathrm{d}}d_{\mathrm{p}}^2)$, where $f(Re_{\mathrm{p}}) = 1 + 0.15Re_{\mathrm{p}}^{0.687}$ is a correction function of the particle Reynolds number, $Re_{\mathrm{p}} = \rho \mid \boldsymbol{U} - \boldsymbol{V} \mid d_{\mathrm{p}}/\mu$.

The volumetric forces take into account the weight and the buoyancy: $f_j^V = \rho g_j$ and $f_j^{\mathrm{d}V} = \alpha^{\mathrm{d}}(\rho^{\mathrm{d}} - \rho)g_j$, where $\boldsymbol{g}$ is the gravity.

The turbulence of the continuous phase is modelled following the k–ϵ strategies, whilst the Eulerian dispersed phase equations are obtained as the statistical moments of a postulated Boltzmann-like evolution equation for the dispersed elements PDF.

The two evolution equation for k and ϵ for the gas phase are solved in order to obtain the gas eddy viscosity $\mu_T = \rho C_\mu k/\epsilon$. σ_k and σ_ϵ are the respective Schmidt numbers. $\mathscr{P}$ is the standard production term also found in single phase flow. The constants C_μ, C_{ϵ_1}, C_{ϵ_2}, σ_k and σ_ϵ have the same values as in single-phase flow.

The interaction-modulation terms I^W, I^{WD} and I^ϵ provide information about the presence of the second phase. I^W, sometimes called redistribution term, expresses the fluctuating work interchanged between the two phases and is written as

$$I^W = C_D\alpha^{\mathrm{d}}\rho^{\mathrm{d}}\left(\overline{u_i'v_i'}^{\mathrm{d}} - \overline{v_i'v_i'}^{\mathrm{d}}\right).$$

The proposed closure for I^W is (Aliod and Dopazo, 1990)

$$I^W \approx C_D\alpha^{\mathrm{d}}\rho^{\mathrm{d}}\left(k\theta - k^d\right); \quad \theta = \frac{\tau_{\mathrm{L}}}{\tau_{\mathrm{L}} + C_D^{-1}}; \quad \tau_{\mathrm{L}} = 0.4\frac{k}{\epsilon} \tag{9}$$

The term I^{WD}, following Crowe (Crowe and Gillandt, 1998), reflects the conversion of mechanical work by the drag force into turbulent kinetic energy. It is expressed as (sum is understood in the index $i = 1, 2, 3$)

$$I^{WD} = C_D\alpha^{\mathrm{d}}\rho^{\mathrm{d}}[U_i - V_i]^2. \tag{10}$$

The closure for I^ϵ can be written as (Aliod and Dopazo, 1990)

$$I^\epsilon \approx \alpha^{\mathrm{d}}\rho^{\mathrm{d}}C_D C_\epsilon \frac{\epsilon}{k}\left\{[U_i - V_i]^2 - \left[k\theta - k^{\mathrm{d}}\right]\right\}.$$

It includes explicitly the effect of the particle's wake through the presence of the square of relative velocity and a redistribution energy term between the phases. C_ϵ is a coefficient with the value 1.15 adopted after a calibration with a series of experiments (Mostafa et al., 1989; Hishida and Maeda, 1997; Modarres et al., 1984).

3. Dispersed phase turbulence

Although particle fluctuating transport $\overline{\phi' v_j'}^{\mathrm{d}}$ (here ϕ' denotes any generic variable of the discrete elements) can not exclusively be related, in general, to the mean velocity shear (Boussinesq–Prandtl hypothesis), for the limit of large inertial particles in simple shear flows, Reeks' theoretical work (Reeks, 1993) shows that the Boussinesq–Prandtl hypothesis is feasible for momentum transport. In this work, Reeks split up the particle Reynolds stresses in two components: A homogeneous component, whose structure is the same as if the local carrier flow were homogeneous, and a deviatoric component involving terms proportional to the mean shear of both the dispersed and carrier flows. However, for long particle response times the deviatoric component of the shear stresses dominate over the homogeneous component reaching a finite value of $-\frac{1}{2}\epsilon_\infty S^{\mathrm{d}}$, where ϵ_∞ is the long-time particle diffusion coefficient in the transverse direction and S^{d} the shear gradient of the dispersed phase. In addition, in this limit the diffusivity momentum coefficient, μ^{d}, is said to be proportional to ϵ_∞. Also, in this case of great inertia particles, Reeks shows that the normal stresses in the streamwise direction are enhanced over the corresponding ones in the other directions and that the former prevail over the shear stresses.

According to the previous considerations, the following closure of the Reynolds stresses is proposed (Laín, 1997):

$$-\rho^{\mathrm{d}}\overline{v_i'v_j'}^{\mathrm{d}} = -\frac{2}{3}\delta_{ij}\left(C^{(i)}\rho^{\mathrm{d}}k^{\mathrm{d}} + \mu^{\mathrm{d}}V_{k,k}\right) + \mu^{\mathrm{d}}\left[V_{i,j} + V_{j,i}\right]. \tag{11}$$

Recalling that the long term diffusion coefficient ϵ_∞ is usually written as the product of the particle mean square velocity and its response time, the dispersed phase turbulent viscosity is defined as

$$\mu^{\mathrm{d}} = C_{\mathrm{p}}\rho^{\mathrm{d}}k^{\mathrm{d}}C_D^{-1}. \tag{12}$$

C_{p} is a coefficient retaining the fact that, on the one hand, the general shear flows of interest are far from satisfying the constant shear and/or homogeneity and, on the other, that the solids have finite inertia. For the case of great inertia particles (but not infinity) a value of the order 10^{-2} for C_{p} can be devised and has been adopted after comparison with a set of experiments.

The anisotropy parameters $C^{(i)}$, $i = 1, 2, 3$, are present because in a general two-phase flow the dispersed elements velocity fluctuations can be far from isotropy. In fact, in the configuration of axisymmetric particle-laden jets, considering the data of Mostafa et al. (1989), Hishida and Maeda (1997), Modarres et al. (1984), an estimation for $C^{(i)}$ can be performed: $C^{(x)} = 2.6$; $C^{(r)} = C^{(\varphi)} = 0.2$ ((x, r, φ) are, respectively, the axial, radial and azimuthal coordinates in the jet).

Consequently, the turbulent transport of k^{d} in the right-hand side of (7) is expressed as

$$-\rho^{\mathrm{d}}\overline{k'^{\mathrm{d}}v_j'}^{\mathrm{d}} = \frac{\mu^{\mathrm{d}}}{\sigma_k^{\mathrm{d}}}k_{,j}^{\mathrm{d}}. \tag{13}$$

Here, σ_k^{d} is the turbulent Schmidt number for k^{d}, which was assigned to 0.3 in the present work.

Finally, the contribution $\mathscr{P}^{\mathrm{d}} = -\rho^{\mathrm{d}}\overline{v_i'v_j'}^{\mathrm{d}}V_{i,j}$ is an *intrinsic* production term analogous to that appearing in the continuous phase Eq. (3).

The numerical values of C_{p} and σ_k^{d} can be compared with other references that make use of a evolution equation for the turbulent kinetic energy of the dispersed phase. As an example, (Huang and Zhou, 1990) working with three-dimensional turbulent recirculating gas–particle flows, report $C_{\mathrm{p}} = 0.064$ and $\sigma_k^{\mathrm{d}} = 0.35$, but in their work the diffusivity μ^{d} is related not only with k^{d} but also with k, which can explain such a difference. On the other hand, σ_k^{d} values are very similar in both works.

4. Analysis and discussion

4.1. Analysis of the equations for the dispersed phase

The non-uniform configuration considered in this study has been the unconfined axisymmetric turbulent particle-laden jet of Mostafa et al. (1989), which is briefly described below.

The transport equations given in Section 2 have been solved by an elliptic finite-volume method. This method as well as the computational domain and boundary conditions employed are described in detail in Laín (1997). The particle volume-fraction at the inlet was approximately 1.0×10^{-4}, thus justifying the absence of inter-particle collisions in the model, and the mass loading $\chi = 0.2$, defined as the ratio of particle-to-gas mass flow rate at the inlet plane.

In a first stage, we are interested in the underlying mechanisms that rule out in the continuum equations describing the particulate phase. To analyse them, Eqs. (2)–(4), (6) and (7) are divided in four global contributions

$$\text{Convection} = \text{Diffusion} + \text{Source} + \text{Interaction}. \tag{14}$$

Here, the sources have been split in two categories. On the one hand the terms denoted by I^x in the system (1)–(7), which constitute the so-called *Interaction* contribution, and, on the other, the rest of source terms, grouped in the denomination *Source*.

The interaction terms in the continuous phase add only a modulation to the present equilibrium in single phase flow. This behaviour could have been expected because the interaction terms are proportional to α^{d} and the flow considered is very dilute. Therefore, such equilibriums for the continuous phase variables will not be showed here. The situation for the solid phase needs to be considered apart. Snapshots for the terms defined in (14) for a representative section of the jet ($X/D = 12.45$, where X is the axial coordinate downstream the nozzle and D is its diameter) are shown in Figs. 1–3.

The main character shown by the equation of axial momentum, Fig. 1, is the convection–interaction equilibrium, since the diffusion and source contributions are pretty small compared with the former ones. Apparently, the convection terms, representing the acceleration, are equal to those forces responsible for the interchange of momentum between the phases.

More information can be extracted from the inspection of the radial momentum equation plot, Fig. 2. Beyond the zone near of the symmetry axis, the most relevant contributions are the source and interaction terms, modulated by the convection

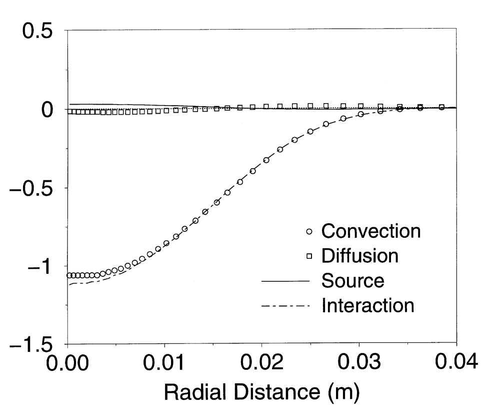

Fig. 1. Snapshot of the relative weight of the terms present in the equation for dispersed phase axial velocity, [m s^{-2}], in a typical axial station of the experiments of Mostafa et al. (1989).

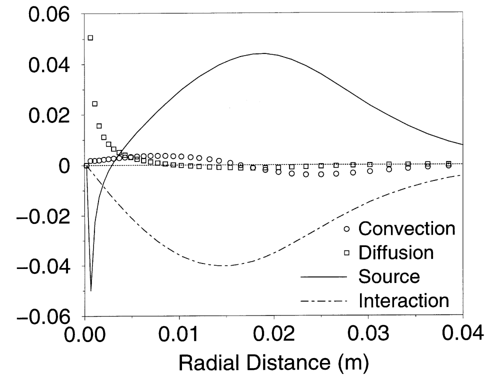

Fig. 2. Snapshot of the relative weight of the terms present in the equation for dispersed phase radial velocity, [m s^{-2}], in a typical axial station of the experiments of Mostafa et al. (1989).

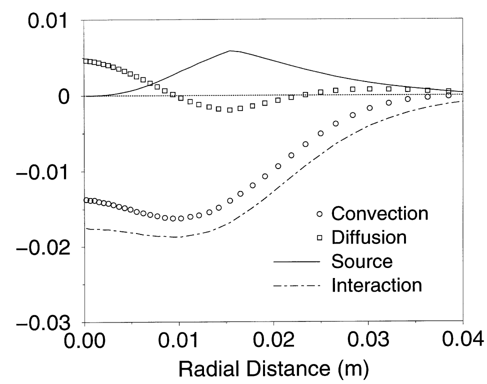

Fig. 3. Snapshot of the relative weight of the terms present in the equation for dispersed phase fluctuating kinetic energy, [m s^{-3}], in a typical axial station of the experiments of Mostafa et al. (1989).

term. Looking closer to the structure of each term, the following approximate equilibrium can be established:

$$\alpha^{\mathrm{d}} C_D W_{\mathrm{r}} \approx \frac{2}{3} C^{(r)} k^{\mathrm{d}} \alpha^{\mathrm{d}}_{,r}. \tag{15}$$

Here, $W_{\mathrm{r}} = U_{\mathrm{r}} - V_{\mathrm{r}}$ is the radial relative velocity. From (15) an expression for W_{r} is easily found by using (12):

$$W_{\mathrm{r}} \propto \frac{\mu^{\mathrm{d}}}{\rho^{\mathrm{d}}} \frac{\alpha^{\mathrm{d}}_{,r}}{\alpha^{d}} \tag{16}$$

(16) is directly related to other closures appeared in the literature (Ishii, 1975,Wiesler, 1986 and more recently Reeks, 1993, Aliod and Dopazo, 1990, Simonin, 1990). In Section 4.2 we show how this behaviour implies the spreading of the α^{d} profile along the jet.

It is necessary to point out that the peaks of the diffusion and source contributions near the symmetry axis are only due to the inclusion into the sources of the extra diffusion term $-2\mu^{\mathrm{d}} V_{\mathrm{r}}/r$ that appears in cylindrical coordinates.

In the k^{d} equation the behaviour of the different contributions is similar to the axial momentum ones. The convection term is roughly compensated with the interaction term, while the diffusion and source terms modulate this balance. Therefore, the change in the turbulent kinetic energy is mainly due to the interchange of fluctuating work between the phases. The redistribution term is negative, which means that there is a transfer of fluctuating energy from the particles to the fluid.

This feature was also found by Wang et al. (1997) in a channel flow configuration working with Large Eddy Simulation. Finally, it must be mentioned that the source term corresponds to the specific production term for the dispersed phase, $\mathscr{P}^{\mathrm{d}}$, which is small with respect to the dominant terms, as it follows from the associated plot in Fig. 3.

4.2. Relationship between the radial relative velocity and the evolution of α^{d} profile along the jet

To illustrate this fact, it is necessary to perform the following change of variables in the equations system (1)–(7):

$$\rho^m = \alpha\rho + \alpha^{\mathrm{d}}\rho^{\mathrm{d}},$$
$$V_i^m = \frac{\alpha\rho U_i + \alpha^{\mathrm{d}}\rho^{\mathrm{d}} V_i}{\rho^m}. \tag{17}$$

Here, ρ^m is the average density of the gas–solid flow, and V_i^m is its mass-weighted velocity. If $W_i = U_i - V_i$ is the relative velocity, it is possible to write: $U_i = V_i^m + \alpha^{\mathrm{d}}\rho^{\mathrm{d}} W_i/\rho^m$ and $V_i = V_i^m - \alpha\rho W_i/\rho^m$. Therefore, introducing these expressions in (5) the following equation for α^{d}, in the stationary case and for constant ρ and ρ^{d}, is obtained:

$$\rho^m V_i^m \rho^{\mathrm{d}} \alpha^{\mathrm{d}}_{,i} = \left[\rho\alpha\rho^{\mathrm{d}}\alpha^{\mathrm{d}} W_i\right]_{,i} + \rho^m_{,i}\rho^{\mathrm{d}}\alpha^{\mathrm{d}} V_i. \tag{18}$$

$(\rho^m V_i^m)_{,i} = 0$, which results from the addition of the stationary Eqs. (1) and (5) and the continuity constraints.

Substituting the expression for radial relative velocity (16) in (18) a diffusive term proportional to $(\rho\alpha\mu^{\mathrm{d}}\alpha^{\mathrm{d}}_{,r})_{,r}$ is found. This contribution, in axisymmetric jet flows, is much more important than the corresponding one implying derivatives in the axial direction and it is directly responsible for the spreading of dispersed phase volume fraction profile along the jet.

It can be summarized, from the analysis of the contributions in the equations for the dispersed phase, that *the modelling of the interaction between both phases is essential as long as they drive the behaviour as a continuum of the discrete elements*. Moreover, the expression of the particle normal stresses, making use of the anisotropy parameters, is directly related to the spreading of the α^{d} – volume fraction distribution along the jet by means of introducing an explicit diffusion term in the continuity equation for the dispersed phase, which confirms several closures used previously for the radial relative velocity. As an additional point, the modelling (11) provides the correct prediction of the evolution of the α^{d} profile in the experiments of Mostafa et al. (1989), which will be shown next.

5. Comparison with experiments

In this section the obtained results employing Eqs. (1)–(7), including the corresponding closures are compared with the experiments of Mostafa et al. (1989).

The experimental configuration was characterized by an air jet laden with glass particles flowing downwards, without inlet swirl, issuing from a 25.3 mm diameter into a 457 mm-square cage assembly. Data were obtained at several axial positions: 15, 25, 35, 50, 75, 150 and 300 mm from the exit plane. Besides, the data at the exit plane served as initial conditions for the numerical calculation.

Because the authors are interested in the application to gas turbines, the majority of the test sections are located close to nozzle, where the turbulent jet is not completely developed. For this reason, only the last two stations will be used to compare with the results of the model (1)–(7).

The calculations together with the experimental data are presented in Figs. 4 and 5 for the cross-sections 150 and 300 mm downstream the nozzle, corresponding to the ratios $X/D = 6.2$ and 12.45. All the flow quantities are plotted versus radial distance in absolute values in order to get a real feeling

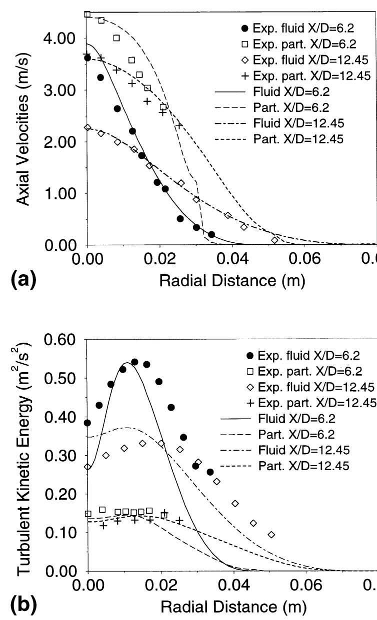

Fig. 4. (a) Axial velocity and (b) fluctuating kinetic energy radial profiles for Mostafa et al. (1989) in two axial stations.

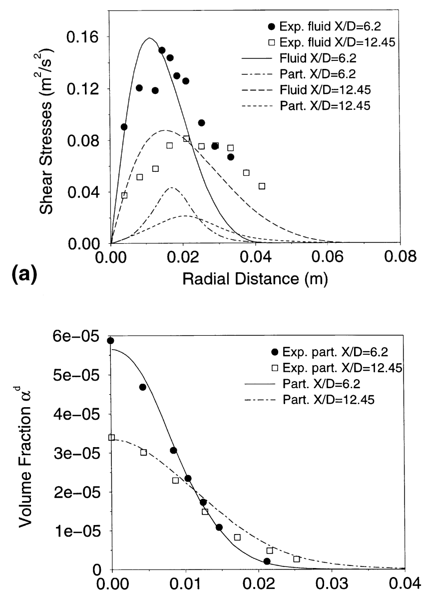

Fig. 5. (a) Shear stresses and (b) volume fraction radial profiles for Mostafa et al. (1989) in two axial stations. The calculated data of particle shear have been scaled by a factor of three in order to get a higher resolution in the same plot than the gas shear stresses.

of the quantitative quality of the predictions. Only the calculated particle shear stresses, for which no experimental data are provided, are scaled by a factor three in order to be shown in the same plot as the gas shear stresses.

In Figs. 4 and 5, a reasonable good agreement between calculations and measurements in all available mean quantities can be observed. The centerline values and the shape of the axial velocity profiles are well captured, especially in the cross-section far downstream, Fig. 4. The calculated maximum values for the turbulent kinetic energy (Fig. 4(b)) and shear stress (Fig. 5) of the gas phase are close to the experimental values, but a little bit displaced to the symmetry axis. This fact has been related to a slight overprediction of the dissipation rate (Laín, 1997). It is stressed that the good results obtained for the dispersed phase fluctuating kinetic energy are due to the existence of an own transport equation for this variable, (7), with a specific production contribution. Special attention is directed to the satisfactory agreement in the solid volume fraction profiles (Fig. 5), where the anisotropy parameters appearing in the expression of the normal stresses in the closure (11) have played a crucial role.

In Fig. 6(a) the normal components of the Reynolds stresses are compared with the referred values provided for the experiment of Mostafa et al. (1989). The same is shown in Fig. 6(b) for another experiment performed by Hishida and Maeda in a gas–solids jet with coflow. The details of the experimental set-up are given by Hishida and Maeda (1997). In both diagrams the same values for the anisotropy parameters presented in Section 3 are used. A reasonable agreement is found in both cases, especially in the sections far downstream, where the jet is completely developed. This concerns both, the axial and radial particle velocity fluctuations.

The next step would be the prediction of this observed anisotropy by means of particle Reynolds stress models, a task that is currently under development.

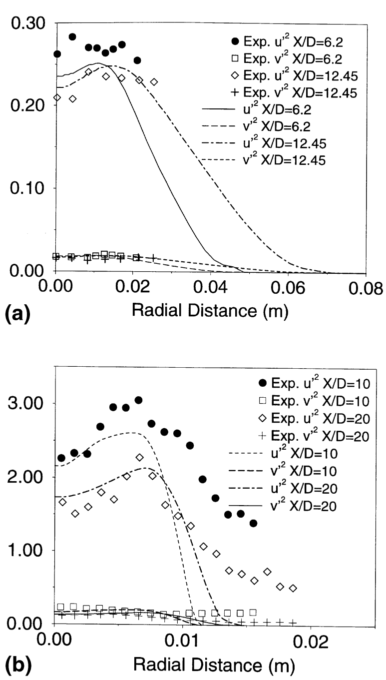

Fig. 6. Normal particle Reynolds stresses, [m^2 s^{-2}] for the experiment of Mostafa et al. (1989) (a) and Hishida and Maeda (1997) (b) based on formula (10). u'^2 corresponds to the axial streamwise direction and v'^2 to the transversal direction.

6. Conclusions

A study on the continuum equations that describe the particulate phase in a non-uniform dilute turbulent gas–solid jet has been performed. It is concluded, that in the case of great inertia particles the modelling of discrete elements normal stresses drives the spreading of the solid volume fraction along the jet. If a consistent expression is given for the former, based on the anisotropy parameters, a confirmation of several closures previously used for the radial relative velocity is found. Moreover, this work reveals the importance of the formulation of the interaction terms (i.e., forces and fluctuating work exchanged between both phases) for achieving the correct evolution of the particle mean quantities involved.

As a matter of future research, several dispersed phase Reynolds stress models will be tried in order to predict the previously cited anisotropy.

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